# A New Iterative Algorithm for Estimating Parameters and Orders of Multiple-Input Single-Output Time Series Models

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**Research Article**

**Abstract**

In this paper, we propose a new iterative algorithm for estimating the parameters and orders of a multiple-input single-output (MISO) time series model. This algorithm is based on a method suggested by Hannan and Rissanen (1982) for estimating an ARMA model. The key is the use of pseudo-linear regression techniques to derive the iterative nonlinear least-squares estimators by using the Gauss-Newton algorithm. Simulation results are presented to compare the new algorithm with the exact maximum likelihood method (EML) and the generalized least squares (GLS) method proposed by Sabiti (1997).

**Keywords:** Nonlinear estimation, time series models, multiple-input single-output models, Gauss-Newton algorithm, recursive estimation

## 1. Introduction

For the treatment of the single-input single-output (SISO) models and the multiple-input single-output (MISO) models, several recent papers have discussed either the asymptotic Fisher information matrix (Klein and Mélard (1994b) and Klein and Mélard (2004)) or the exact Fisher information matrix (Klein, Mélard and Zahaf (1998), Klein and Mélard (1994a) and Zadrozny (1989, 1992)) but we have seen no indication of the approach used to estimate the parameters of these models. Ouakasse and Mélard (2014) have proposed a complicated recursive estimation method for SISO models where the recurrence for updating the Hessian is avoided but the recurrence for updating the estimator makes use of the Fisher information matrix. In this paper, we describe a simple iterative algorithm for estimating the parameters of a MISO model given by the equation

$$\frac{\alpha(B)}{\lambda(B)}y(t)=\sum_{i=1}^{k}\frac{\omega_i(B)}{\delta_i(B)}x_i(t-b_i)+\frac{\theta(B)}{\phi(B)}\varepsilon(t) \qquad (1)$$

where $y(t)$ is the endogenous variable, $x_i(t)$ $(i=1,...,k)$ are the exogenous variables, $B$ is the backshift operator such that $B^j y(t)=y(t-j)$, $\varepsilon(t)$ are normally and independently random variables with mean zero and constant variance $\sigma_\varepsilon^2$ and $b_i$ is the delay of transmission of influence between the $i$ th exogenous variable and the endogenous variable, or the delay parameter which represents the number of complete time intervals before a change in $x_i(t)$ begins to have an effect on $y(t)$, $\beta=\left(\alpha^T,\lambda^T,\omega_1^T,...,\omega_k^T,\delta_1^T,...,\delta_k^T,\phi^T,\theta^T\right)^T$ is the vector of parameters to be estimated where

$$\alpha=\left(\alpha_1,...,\alpha_{\bar{s}}\right)^T,\ \lambda=\left(\lambda_1,...,\lambda_{\bar{r}}\right)^T,\ \omega_i=\left(\omega_{i0},\omega_{is_1},...,\omega_{is_i}\right)^T \qquad (2a)$$

$$\delta_i=\left(\delta_{ir_1},...,\delta_{ir_i}\right)^T,\ \phi=\left(\phi_1,...,\phi_p\right)^T,\ \theta=\left(\theta_1,...,\theta_q\right)^T \qquad (2b)$$

and $l=\bar{s}+\bar{r}+s_1+...+s_k+r_1+...+r_k+p+q$ is the number of parameters. The different polynomials in (1) are given by

$$\alpha(B)=1-\sum_{j=1}^{\bar{s}}\alpha_j B^j,\quad \lambda(B)=1-\sum_{j=1}^{\bar{r}}\lambda_j B^j,\quad \alpha_0=\lambda_0=1 \qquad (3a)$$

$$\phi(B)=1-\sum_{j=1}^{p}\phi_j B^j,\quad \theta(B)=1-\sum_{j=1}^{q}\theta_j B^j,\quad \phi_0=\theta_0=1 \qquad (3b)$$

$$\omega_i(B)=\omega_{i0}+\sum_{j=1}^{s_i}\omega_{ij}B^j,\quad \delta_i(B)=1-\sum_{j=1}^{r_i}\delta_{ij}B^j,\quad \delta_{i0}=1,\quad i=1,...,k. \qquad (3c)$$

The assumptions (1, 2a and 2b) made on these six polynomials are the regularity conditions to ensure the stability of the model (1), see Sabiti (1997).

The equation (1) can be written under the f

$$\varphi(B)y(t)=\sum_{i=1}^{k}\kappa_i(B)x_i(t-b_i)+\pi(B)\varepsilon(t). \qquad (4)$$

where following Jorgenson (1966), the series $\varphi(B)$, $\kappa_i(B)$ and $\pi(B)$ defined as a ratio of two polynomials such as

$$\varphi(B)=\frac{\alpha(B)}{\lambda(B)},\quad \kappa_i(B)=\frac{\omega_i(B)}{\delta_i(B)},\quad \pi(B)=\frac{\theta(B)}{\phi(B)},\quad i=1,...,k. \qquad (5)$$

We assume that the coefficients $\{\varphi_j\}$, $\{\kappa_{ij}\}$ and $\{\pi_j\}$ form convergent series, i.e.

$$\varphi(B)=1-\sum_{j=1}^{\infty}\varphi_j B^j,\quad \sum_{j=1}^{\infty}\left|\varphi_j\right|\prec\infty,\quad \varphi_0=1,. \qquad (6a)$$

$$\kappa_i(B) = \kappa_{i0} + \sum_{j=1}^{\infty} \kappa_{ij} B^j , \quad \sum_{j=1}^{\infty} \left| \kappa_{ij} \right| \prec \infty, \quad \kappa_{i0} \neq 1, . \tag{6b}$$

$$\pi(B) = 1 - \sum_{j=1}^{\infty} \pi_j B^j , \quad \sum_{j=1}^{\infty} \left| \pi_j \right| \prec \infty, \quad \pi_0 = 1, \quad i = 1,\dots,k . \tag{6c}$$

The series $\kappa_i(B)$ is called the $i$th transfer function of the process in the Box and Jenkins (1976) terminology. When the series $\varphi(B)$, $\kappa_i(B)$ and $\pi(B)$ are written in the form (6a-b), the weights or coefficients series $\varphi_j$, $\kappa_{ij}$ and $\pi_j$ are referred to as the impulse response of this time series model.

**2. The Hannan and Rissanen (1982) method**

When $\bar{s} = \bar{r} = s_i = r_i = 0$, the model (1) is reduced to an ARMA model treated by Hannan and Rissanen (1982) and given by

$$y(t) = \frac{\theta(B)}{\phi(B)} \varepsilon(t) . \tag{7a}$$

The model ARMA (7) can also be written in the form

$$y(t) = \phi_1 y(t-1) + \dots + \phi_p y(t-p) - \theta_1 \varepsilon(t-1) - \dots - \theta_q \varepsilon(t-q) + \varepsilon(t) . \tag{7b}$$

The Hannan and Rissanen (1982) method for estimating the parameter vector $\beta = \left(\phi^T, \theta^T\right)^T$ of the equation (7b) can be divided into the three following stages.

**Stage 1** : An autoregression of order $h$ of the form

$$y(t) = \sum_{j=1}^{h} \pi_j y(t-j) + \varepsilon(t) . \tag{8a}$$

is estimated and the autoregressive coefficients $\hat{\pi}_j$ $(j = 1,\dots,h)$ are used to estimate the sample innovations $\hat{\varepsilon}(t)$ as

$$\hat{\varepsilon}(t) = y(t) - \sum_{j=1}^{h} \hat{\pi}_j y(t-j) . \tag{8b}$$

The order $h$ may be selected by minimizing the information criterion

$$BIC(h) = \log \hat{\sigma}_h^2 + \frac{h \log N}{N} . \tag{9}$$

Put $J = 0$ where $J$ is the iteration counter and proceed with stage 2.

**Stage 2** : The innovations estimators $\hat{\varepsilon}(t)$ are used in the regression

$$y(t) = \phi_1 y(t-1) + \dots + \phi_p y(t-p) - \theta_1 \hat{\varepsilon}(t-1) - \dots - \theta_q \hat{\varepsilon}(t-q) + \zeta(t) \tag{10}$$

which is fitted by the ordinary least squares (OLS) method for obtaining the initial estimators $\tilde{\phi}_j$ and $\tilde{\theta}_j$ of the true parameters $\phi_j$ and $\theta_j$. In that second stage, Hannan and Rissanen (1982) determine the model orders by evaluating the variance for each $(p,q)$ from

$$\tilde{\sigma}^2_{(p,q)} = \inf \frac{1}{N}\sum_{t=\ell}^{N}\left\{y(t) - \sum_{j=1}^{p}\tilde{\phi}_j^{(J)} y(t-j) + \sum_{j=1}^{q}\tilde{\theta}_j^{(J)}\hat{\varepsilon}(t-j)\right\}^2 \qquad (11)$$

where $\ell = \max(h+p+1, h+q+1)$. Then the orders $(\tilde{p},\tilde{q})$ are selected to minimize the information criterion

$$BIC(p,q) = \log\hat{\sigma}^2_{(p,q)} + (p+q)\frac{\log N}{N}. \qquad (12)$$

The initial estimators $\tilde{\phi}_j^{(J)}$ and $\tilde{\theta}_j^{(J)}$ that minimize (11) for $(p,q) = (\tilde{p},\tilde{q})$ are consistent but not asymptotically efficient. Since the orders $(\tilde{p},\tilde{q})$ are known, Hannan and Rissanen (1982) have proposed the use of a simple optimization algorithm.

**Stage 3** : In the third stage, Hannan and Rissanen (1982) use the consistent estimators $\tilde{\phi}_j^{(J)}$ and $\tilde{\theta}_j^{(J)}$ as initial values in the Gauss-Newton iterations optimizing a Gaussian likelihood function. This third stage consists in forming the following equations

$$\tilde{\varepsilon}(t) = y(t) - \sum_{j=1}^{\tilde{p}}\tilde{\phi}_j^{(J)} y(t-j) + \sum_{j=1}^{\tilde{q}}\tilde{\theta}_j^{(J)}\tilde{\varepsilon}(t-j) \qquad (13a)$$

$$u(t) = \sum_{j=1}^{\tilde{p}}\tilde{\phi}_j^{(J)} u(t-j) + \tilde{\varepsilon}(t), \quad v(t) = \sum_{j=1}^{\tilde{q}}\tilde{\theta}_j^{(J)} v(t-j) + \tilde{\varepsilon}(t). \qquad (13b)$$

After obtaining the series $\tilde{\varepsilon}(t)$, $u(t)$ and $v(t)$, they have fitted the OLS method the following regression

$$\tilde{\varepsilon}(t) = \sum_{j=1}^{\tilde{p}}\Delta\tilde{\phi}_j^{(J)} u(t-j) - \sum_{j=1}^{\tilde{q}}\Delta\tilde{\theta}_j^{(J)} v(t-j) + \varepsilon(t) \qquad (14)$$

for obtaining the regression coefficients $\Delta\tilde{\phi}_j^{(J)}$ and $\Delta\tilde{\theta}_j^{(J)}$ which, when they are added respectively to $\tilde{\phi}_j^{(J)}$ and $\tilde{\theta}_j^{(J)}$, give the estimators

$$\hat{\phi}_j^{(J+1)} = \tilde{\phi}_j^{(J)} + \Delta\hat{\phi}_j^{(J)}, \quad j = 1,\ldots,\tilde{p} \qquad (15a)$$

$$\hat{\theta}_j^{(J+1)} = \tilde{\theta}_j^{(J)} + \Delta\hat{\theta}_j^{(J)}, \quad j = 1,\ldots,\tilde{q} \qquad (15b)$$

that have the same asymptotic distribution as maximum likelihood estimators. That procedure can be iterated starting from $\hat{\phi}_j^{(J+1)}$ and $\hat{\theta}_j^{(J+1)}$ instead of the $\tilde{\phi}_j^{(J)}$ and $\tilde{\theta}_j^{(J)}$ at (13) until $\hat{\sigma}^2_{\hat{\varepsilon}}$ becomes stable.

## 3. The pseudo-linear regression form of a MISO model

For obtaining the pseudo-linear regression form of a MISO model, the approach consists to write the MISO model (1) as

$$\varphi(B)y(t) = \sum_{i=1}^{k}\kappa_i(B)x_i(t-b_i) + e(t). \qquad (16a)$$

where the coefficients $\varphi_j$ and $\kappa_{ij}$ are defined in (6a) and (6b). The equation (16) can be approximated by a regression of order $h$ sufficiently large given by

$$y(t)=\sum_{j=1}^{h}\varphi_j y(t-j)+\sum_{i=1}^{k}\sum_{j=0}^{h}\kappa_{ij}x_i(t-b_i-j)+e(t)\,. \qquad (16b)$$

where the error $e(t)$ is written as an ARMA$(p,q)$ model

$$e(t)=\frac{\theta(B)}{\phi(B)}\varepsilon(t). \qquad (17)$$

We can also write the MISO model (1) as

$$\frac{\alpha(B)\phi(B)}{\lambda(B)\theta(B)}y(t)=\sum_{i=1}^{k}\frac{\omega_i(B)\phi(B)}{\delta_i(B)\theta(B)}x_i(t-b_i)+\varepsilon(t) \qquad (18)$$

or in the form

$$\Lambda(B)y(t)=\sum_{i=1}^{k}\Psi_i(B)x_i(t-b_i)+\varepsilon(t) \qquad (19)$$

where the coefficients $\Lambda_j$ and $\Psi_{ij}$ are defined respectively as

$$\frac{\alpha(B)\phi(B)}{\lambda(B)\theta(B)}=\Lambda(B)=1-\sum_{j=1}^{\infty}\Lambda_j B^j\,,\quad \sum_{j=1}^{\infty}\left|\Lambda_j\right|\prec\infty \qquad (20a)$$

$$\frac{\omega_i(B)\phi(B)}{\delta_i(B)\theta(B)}=\Psi_i(B)=\sum_{j=0}^{\infty}\Psi_{ij}B^j\,,\quad \sum_{j=0}^{\infty}\left|\Psi_{ij}\right|\prec\infty,\quad i=1,\ldots,k\,. \qquad (20b)$$

The equation (17) can also be expressed as an autoregressive representation given by the equation

$$\pi(B)e(t)=\varepsilon(t) \qquad (21)$$

where we suppose that the coefficients of the series $\pi(B)$ form a convergent series, i.e.

$$\frac{\phi(B)}{\theta(B)}=\pi(B)=1-\sum_{j=1}^{\infty}\pi_j B^j\,,\quad \sum_{j=1}^{\infty}\left|\pi_j\right|\prec\infty\,. \qquad (22)$$

A pseudo-linear regression model in the parameters $\alpha_j$, $\lambda_j$, $\omega_{ij}$ and $\delta_{ij}$ can be obtained by starting from

$$\alpha(B)\frac{\phi(B)}{\theta(B)}y(t)=\lambda(B)\frac{\alpha(B)\phi(B)}{\lambda(B)\theta(B)}y(t) \qquad (23a)$$

$$\omega_i(B)\frac{\phi(B)}{\theta(B)}x_i(t)=\delta_i(B)\frac{\omega_i(B)\phi(B)}{\delta_i(B)\theta(B)}x_i(t),\quad i=1,\ldots,k\,. \qquad (23b)$$

The relations (23a-b) can also be written respectively as

$$\alpha(B)[\pi(B)y(t)]=\lambda(B)[\Lambda(B)y(t)] \quad (24a)$$

$$\omega_i(B)[\pi(B)x_i(t)]=\delta_i(B)[\Psi_i(B)x_i(t)], \quad i=1,...,k. \quad (24b)$$

The equation (16a) can now be written in a pseudo-linear form in the parameters $\alpha_j$, $\lambda_j$, $\omega_{ij}$ and $\delta_{ij}$ as follows. By adding the left hand side of the relations (24a) and (24b) respectively in the left and right hand sides of (16a) and by subtracting the right hand side of the relations (24a) and (24b) respectively in the left and the right hand sides of (16a), the equation (16a) is written under the form

$$\alpha(B)[\pi(B)y(t)]-\lambda(B)[\Lambda(B)y(t)]+\Lambda(B)y(t)=\sum_{i=1}^{k}\{\omega_i(B)[\pi(B)x_i(t-b_i)]\}$$

$$-\sum_{i=1}^{k}\{\delta_i(B)[\Psi_i(B)x_i(t-b_i)]\}+\Psi_i(B)x_i(t-b_i)+\varepsilon(t) \quad (25a)$$

or as

$$\alpha(B)[\pi(B)y(t)]-\left(1-\sum_{j=1}^{\bar{r}}\lambda_j B^j\right)[\Lambda(B)y(t)]+\Lambda(B)y(t)=\sum_{i=1}^{k}\{\omega_i(B)[\pi(B)x_i(t-b_i)]\}$$

$$-\sum_{i=1}^{k}\left\{\left(1-\sum_{j=1}^{r_i}\delta_{ij}B^j\right)[\Psi_i(B)x_i(t-b_i)]\right\}+\Psi_i(B)x_i(t-b_i)+\varepsilon(t). \quad (25b)$$

Equivalently, the relation (25b) can also be written as

$$\alpha(B)[\pi(B)y(t)]+\sum_{j=1}^{\bar{r}}\lambda_j[\Lambda(B)y(t-j)]=\sum_{i=1}^{k}\{\omega_i(B)[\pi(B)x_i(t-b_i)]\}$$

$$+\sum_{i=1}^{k}\sum_{j=1}^{r_i}\delta_{ij}[\Psi_i(B)x_i(t-b_i-j)]+\varepsilon(t). \quad (25c)$$

If the estimators of the coefficients $\pi_j$, $\Lambda_j$ and $\Psi_{ij}$ were known, we could generated the variables $\bar{y}(t)$, $\tilde{y}(t)$, $\tilde{x}_i(t)$ and $\bar{x}_i(t)$ from

$$\tilde{y}(t)=\pi(B)y(t), \quad \bar{y}(t)=\Lambda(B)y(t) \quad (26a)$$

$$\tilde{x}_i(t)=\pi(B)x_i(t), \quad \bar{x}_i(t)=\Psi_i(B)x_i(t), \quad i=1,...,k \quad (26b)$$

where we replace

$$\pi(B)=1-\sum_{j=1}^{\infty}\pi_j B^j, \quad \Lambda(B)=1-\sum_{j=1}^{\infty}\Lambda_j B^j, \qquad \Psi_i(B)=\Psi_{ij}+\sum_{j=1}^{\infty}\Psi_{ij}B^j. \quad (27a)$$

respectively by

$$\pi(B)=1-\sum_{j=1}^{\hbar}\pi_j B^j, \quad \Lambda(B)=1-\sum_{j=1}^{h}\Lambda_j B^j, \qquad \Psi_i(B)=\Psi_{ij}+\sum_{j=1}^{h}\Psi_{ij}B^j. \quad (27b)$$

The equation (21) can thus be written as an autoregressive model of the form

$$e(t)=\sum_{j=1}^{h}\pi_j e(t-j)+\varepsilon(t) \qquad (28)$$

where we assume that the approximation error in (27b) and (28) can be made negligible by taking the orders $h$ and $\hbar$ sufficiently large. The transformed variables $\bar{y}(t)$, $\tilde{y}(t)$, $\tilde{x}_i(t)$ and $\bar{x}_i(t)$ are written as

$$\tilde{y}(t)=y(t)-\sum_{j=1}^{\hbar}\pi_j y(t-j), \quad \bar{y}(t)=y(t)-\sum_{j=1}^{\hbar}\Lambda_j y(t-j) \qquad (29a)$$

$$\tilde{x}_i(t)=x_i(t)-\sum_{j=1}^{\hbar}\pi_j x_i(t-j), \quad \bar{x}_i(t)=\sum_{j=0}^{h}\Psi_{ij}x_i(t-j), \quad i=1,\ldots,k\,. \qquad (29b)$$

The equation (25c) can thus be expressed in a pseudo-linear form of these parameters as

$$\tilde{y}(t)=\sum_{j=1}^{\bar{s}}\alpha_j\tilde{y}(t-j)-\sum_{j=1}^{\bar{r}}\lambda_j\bar{y}(t-j)+\sum_{i=1}^{k}\sum_{j=0}^{s_i}\omega_{ij}\tilde{x}_i(t-b_i-j)$$

$$+\sum_{i=1}^{k}\sum_{j=0}^{h}\delta_{ij}\bar{x}_i(t-b_i-j)+e(t)\,. \qquad (30)$$

which can be used to obtain the initial estimators of the parameters $\alpha_j$, $\lambda_j$, $\omega_{ij}$, $\delta_{ij}$ and the estimators of $\varepsilon(t)$.

The estimators of the parameters $\alpha_j$, $\lambda_j$, $\omega_{ij}$ and $\delta_{ij}$ can be used a second time to evaluate the residuals $e(t)$ from

$$e(t)=\frac{\alpha(B)}{\lambda(B)}y(t)-\sum_{i=1}^{k}\frac{\omega_i(B)}{\delta_i(B)}x_i(t-b_i) \qquad (31)$$

and the regression

$$e(t)=\sum_{j=1}^{p}\phi_j e(t-j)-\sum_{j=1}^{q}\theta_j\varepsilon(t-j)+\varepsilon(t) \qquad (32)$$

can thus be fitted for obtaining the estimators of the parameters $\phi_j$ and $\theta_j$.

## 4. Description of the proposed algorithm

For estimating the parameter vector $\beta=\left(\alpha^T,\lambda^T,\omega_1^T,\ldots,\omega_k^T,\delta_1^T,\ldots,\delta_k^T,\phi^T,\theta^T\right)^T$ of the MISO model (1), the proposed algorithm can be divided into the following stages.

**Stage 1**: The regression (16b) given by the following equation is fitted by the ordinary least squares (OLS) method for obtaining the estimators $\hat{\varphi}_j$ and $\hat{\kappa}_{ij}$ of the coefficients $\varphi_j$ and $\kappa_{ij}$. These estimators $\hat{\varphi}_j$ and $\hat{\kappa}_{ij}$ are used to estimate the residuals $\hat{e}(t)$ as

$$\hat{e}(t)=y(t)-\sum_{j=1}^{h}\hat{\varphi}_j y(t-j)-\sum_{i=1}^{k}\sum_{j=0}^{h}\hat{\kappa}_{ij}x_i(t-b_i-j)\,. \qquad (33)$$

The order $h$ may be selected by minimizing the information criterion

$$BIC(h) = \log \hat{\sigma}_{h}^{2} + h\frac{\log N}{N} \,.. \qquad (34)$$

Put $J = 0$ where $J$ is the iteration counter and proceed with the stage 2.

**Stage 2** : Fit the autoregression (28) of order $\hbar$ by regressing $\hat{e}(t)$ on $\hat{e}(t-j)$ $(j = 1,...,\hbar)$ for obtaining the coefficients $\hat{\pi}_j$ and innovations estimators $\hat{\varepsilon}(t)$. These innovations $\hat{\varepsilon}(t)$ are obtained from

$$BIC(\hbar) = \log \hat{\sigma}_{\hbar}^{2} + \hbar\frac{\log N}{N} \; \hat{\varepsilon}(t) = \hat{e}(t) - \sum_{j=1}^{h} \hat{\pi}_j e(t-j). \qquad (35)$$

The order $\hbar$ may be selected by minimizing the information criterion (36)

$$BIC(\hbar) = \log \hat{\sigma}_{\hbar}^{2} + \hbar\frac{\log N}{N} \,. \qquad (36)$$

Evaluate the coefficients $\hat{\Lambda}_j$ and $\hat{\Psi}_{ij}$ from

$$\hat{\Lambda}(B) = \hat{\varphi}(B)\hat{\pi}(B), \quad \hat{\Psi}(B) = \hat{\kappa}(B)\hat{\pi}(B), \quad i = 1,...,k \,. \qquad (37)$$

**Stage 3** : Since the coefficients $\hat{\pi}_j$, $\hat{\Lambda}_j$ and $\hat{\Psi}_{ij}$ are thus known, generated the variables $\tilde{y}(t)$, $\bar{y}(t)$, $\tilde{x}_i(t)$ and $\bar{x}_i(t)$ given in (26a-b) as

$$\tilde{y}(t) = \hat{\pi}(B)y(t), \quad \bar{y}(t) = \hat{\Lambda}(B)y(t) \qquad (38a)$$

$$\tilde{x}_i(t) = \hat{\pi}(B)x_i(t), \quad \bar{x}_i(t) = \hat{\Psi}_i(B)x_i(t), \quad i = 1,...,k \,. \qquad (38b)$$

The regression (30) is estimated by the OLS method by regressing $\tilde{y}(t)$ on $\tilde{y}(t-j)$ $(j = 1,...,\bar{s})$, $-\bar{y}(t-j)$ $(j = 1,...,\bar{r})$, $\tilde{x}_i(t-b_i-j)$ $(j = 1,...,s_i)$ and $\bar{x}_i(t-b_i-j)$ $(j = 1,...,r_i)$ for obtaining the initial estimators $\tilde{\alpha}_j$, $\tilde{\lambda}_j$, $\tilde{\omega}_{ij}$ and $\tilde{\delta}_{ij}$ of the parameters $\alpha_j$, $\lambda_j$, $\omega_{ij}$ and $\delta_{ij}$ for $(i = 1,...,k)$.

The orders $(\bar{s}, \bar{r}, s_i, r_i, b_i)$ are selected to minimize the information criterion

$$\min_{b_i} \min_{\bar{s},\bar{r},s_i,r_i} BIC(\bar{s}, \bar{r}, s_i, r_i | b_i) = \log \tilde{\sigma}_{\tilde{e}}^{2} + (\bar{s} + \bar{r} + s_i + r_i)\frac{\log(N-b_i)}{N-b_i} \,. \qquad (39)$$

where $0 \leq (\bar{s}, \bar{r}, s_i, r_i) \prec (\log N)^c$ and $0 \prec c \prec \infty$ provides a bound and $\tilde{\sigma}_{\tilde{e}}^{2}$ is obtained from the OLS estimation of (30).

**Stage 4**: The initial estimators $\tilde{\alpha}_j$, $\tilde{\lambda}_j$, $\tilde{\omega}_{ij}$ and $\tilde{\delta}_{ij}$ for $(i = 1,...,k)$ are replaced in (31) to evaluate the residuals $\tilde{e}(t)$ as

$$e(t) = \frac{\tilde{\alpha}(B)}{\tilde{\lambda}(B)}y(t) - \sum_{i=1}^{k}\frac{\tilde{\omega}_i(B)}{\tilde{\delta}_i(B)}x_i(t-b_i). \qquad (40a)$$

Using these residuals and the innovations estimators $\hat{\varepsilon}(t)$ obtained at the second stage, the following ARMA$(p,q)$ model is given by

$$\tilde{e}(t) = \phi_1 \tilde{e}(t-1) + ... + \phi_p \tilde{e}(t-p) - \theta_1 \hat{\varepsilon}(t-1) - ... - \theta_q \hat{\varepsilon}(t-q) + \zeta(t) \qquad \text{(40b)}$$

and estimated by regressing $\tilde{e}(t)$ on $\tilde{e}(t-j)$ $(j=1,...,p)$ and $\hat{\varepsilon}(t-j)$ $(j=1,...,q)$ for obtaining the initial estimators $\tilde{\phi}_j$ and $\tilde{\theta}_j$ of the true parameters $\phi_j$ and $\theta_j$. The order $(p,q)$ of (32) may be selected by minimizing the information criterion

$$BIC(p,q) = \log \hat{\sigma}^2_{(p,q)} + (p+q)\frac{\log N}{N}. \qquad \text{(41)}$$

**Stage 5** : The initial estimators $\tilde{\alpha}_j$, $\tilde{\lambda}_j$, $\tilde{\omega}_{ij}$, $\tilde{\delta}_{ij}$ for $(i=1,...,k)$, $\tilde{\phi}_j$ and $\tilde{\theta}_j$ are used to evaluate the series $\hat{\varepsilon}(t)$, $\hat{u}(t)$, $\hat{v}(t)$, $\hat{\xi}(t)$, $\hat{\zeta}(t)$, $\hat{\eta}_i(t)$ and $\hat{m}_i(t)$ via the generating equations (Sabiti, 1997) as

$$\tilde{\varepsilon}(t) = \tilde{e}(t) - \sum_{j=1}^{p} \tilde{\phi}_j \tilde{e}(t-j) + \sum_{j=1}^{q} \tilde{\theta}_j \tilde{\varepsilon}(t-j) \qquad \text{(42a)}$$

$$\xi(t) = \frac{\tilde{\phi}(B)}{\tilde{\lambda}(B)\tilde{\theta}(B)} y(t), \quad \zeta(t) = \frac{\tilde{\phi}(B)\tilde{\alpha}(B)}{\tilde{\lambda}^2(B)\tilde{\theta}(B)} y(t) \qquad \text{(42b)}$$

$$\eta_i(t) = \frac{\tilde{\phi}(B)}{\tilde{\delta}_i(B)\tilde{\theta}(B)} x_i(t-b_i), \quad m_i(t) = \frac{\tilde{\omega}_i(B)\tilde{\phi}(B)}{\tilde{\delta}_i^2(B)\tilde{\theta}(B)} y(t) \qquad \text{(42c)}$$

$$v(t) = \frac{\tilde{\varepsilon}(t)}{\tilde{\theta}(B)}, \qquad u(t) = \frac{\tilde{\varepsilon}(t)}{\tilde{\phi}(B)}. \qquad \text{(42d)}$$

The Gauss-Newton regression is formed as

$$\tilde{\varepsilon}(t) = \sum_{j=1}^{\bar{s}} \Delta\alpha_j \xi(t-j) - \sum_{j=1}^{\bar{r}} \Delta\lambda_j \zeta(t-j) + \sum_{j=1}^{s_i} \Delta\omega_{ij} \eta_i(t-j) + \sum_{j=1}^{r_i} \Delta\delta_{ij} m_i(t-j)$$

$$+ \sum_{j=1}^{p} \Delta\phi_j u(t-j) + \sum_{j=1}^{q} \Delta\theta_j v(t-j) + \varepsilon(t) \qquad \text{(43)}$$

is estimated by the OLS method for obtaining the regression coefficients that we call $\Delta\hat{\alpha}_j$, $\Delta\hat{\lambda}_j$, $\Delta\hat{\omega}_{ij}$, $\Delta\hat{\delta}_{ij}$ $(i=1,...,k)$, $\Delta\hat{\phi}_j$ and $\Delta\hat{\theta}_j$ when they are added respectively to the initial estimators $\tilde{\alpha}_j$, $\tilde{\lambda}_j$, $\tilde{\omega}_{ij}$, $\tilde{\delta}_{ij}$ for $(i=1,...,k)$, $\tilde{\phi}_j$ and $\tilde{\theta}_j$ give the estimators

$$\hat{\alpha}_j^{(J+1)} = \tilde{\alpha}_j^{(J)} + \Delta\hat{\alpha}_j^{(J)}, \quad j=1,...,\bar{s} \qquad \text{(44a)}$$

$$\hat{\lambda}_j^{(J+1)} = \tilde{\lambda}_j^{(J)} + \Delta\hat{\lambda}_j^{(J)}, \quad j=1,...,\bar{r} \qquad \text{(44b)}$$


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$$\hat{\omega}_{ij}^{(J+1)} = \tilde{\omega}_{ij}^{(J)} + \Delta\hat{\omega}_{ij}^{(J)}, \quad j = 1,\ldots,s_i \tag{44c}$$

$$\hat{\delta}_{ij}^{(J+1)} = \tilde{\delta}_{ij}^{(J)} + \Delta\hat{\delta}_{ij}^{(J)}, \quad j = 1,\ldots,r_i \tag{44d}$$

$$\hat{\phi}_{j}^{(J+1)} = \tilde{\phi}_{j}^{(J)} + \Delta\hat{\phi}_{j}^{(J)}, \quad j = 1,\ldots,p \tag{44e}$$

$$\hat{\theta}_{j}^{(J+1)} = \tilde{\theta}_{j}^{(J)} + \Delta\hat{\theta}_{j}^{(J)}, \quad j = 1,\ldots,q \tag{44f}$$

for $(i = 1,\ldots,k)$. That procedure can be iterated starting from $\hat{\alpha}_j$, $\hat{\lambda}_j$, $\hat{\omega}_{ij}$, $\hat{\delta}_{ij}$ $(i = 1,\ldots,k)$, $\hat{\phi}_j$ and $\hat{\theta}_j$ instead of the initial estimators $\tilde{\alpha}_j$, $\tilde{\lambda}_j$, $\tilde{\omega}_{ij}$, $\tilde{\delta}_{ij}$ for $(i = 1,\ldots,k)$, $\tilde{\phi}_j$ and $\tilde{\theta}_j$ at (42a) until $\hat{\sigma}_{\hat{\varepsilon}}^2$ becomes stable.

**5. Simulation results**

To study the performance of the proposed algorithm which is a nonlinear least squares (NLS) approach, we compare it with the exact maximum likelihood (EML) and the generalized least squares (GLS) methods suggested by Sabiti (1997). We consider a MISO model with a single exogenous variable :

$$\frac{\alpha(B)}{\lambda(B)} y(t) = \frac{\omega(B)}{\delta(B)} x(t) + \frac{\theta(B)}{\phi(B)} \varepsilon(t), \qquad t = 1,2,\ldots,N$$

for which the delay $b$ will be equal to zero. In order, to realize the 10000 simulations, we need exogenous variable for obtaining the corresponding endogenous variables. For that, we use the exogenous variable presented in the study of Grillenzoni (1990). The simulation results that we present are concerned by the transfer function model studied by Grillenzoni (1990) where two economic series have been considered. We define $x(t)$ as the exchange rate between the sterling pound and the US dollar and $y(t)$ the index of wholesale prices in Italy for January 1973 to December 1985 $(N = 156)$. The results of simulations concern a MISO model of the form:

$$\alpha(B) y(t) = \omega(B) x(t) + \frac{1}{\phi(B)} \varepsilon(t).$$

The simulations results for this model are given in the followings tables for $N = 40$, $N = 80$, $N = 120$ and $N = 156$.

The first simulations concern a MISO model of the form

$$(1 - \alpha_1 B) y(t) = (\omega_0 + \omega_1 B) x(t) + \frac{1}{(1 - \phi_1 B)} \varepsilon(t).$$

**Table 1: Simulation results for** $\alpha_1 = 0.4$ , $\omega_0 = 0.3$ , $\omega_1 = 0.02$ , $\phi_1 = 0.5$ , $N = 40$

| Methods | $\bar{\hat{\alpha}}_1$ | $\bar{\hat{\sigma}}(\bar{\hat{\alpha}}_1)$ | $\bar{\hat{\omega}}_0$ | $\bar{\hat{\sigma}}(\bar{\hat{\omega}}_0)$ | $\bar{\hat{\omega}}_1$ | $\bar{\hat{\sigma}}(\bar{\hat{\omega}}_1)$ | $\hat{\phi}_1$ | $\bar{\hat{\sigma}}(\hat{\phi}_1)$ | $\hat{\sigma}^2_{\hat{\varepsilon}}$ |
|---|---|---|---|---|---|---|---|---|---|
| GLS | 0.3455 | 0.0664 | 0.3021 | 0.0104 | 0.0033 | 0.0241 | 0.5651 | 0.0235 | 1.0597 |
| EML | 0.4961 | 0.1750 | 0.3067 | 0.0115 | 0.0933 | 0.0135 | 0.5206 | 0.1819 | 1.7203 |
| NLS | 0.4432 | 0.0566 | 0.3028 | 0.0112 | 0.0189 | 0.0139 | 0.5189 | 0.0243 | 1.2450 |

4.

**Table 2: Simulation results for** $\alpha_1 = 0.4$ , $\omega_0 = 0.3$ , $\omega_1 = 0.02$ , $\phi_1 = 0.5$ , $N = 80$

| Methods | $\bar{\hat{\alpha}}_1$ | $\bar{\hat{\sigma}}(\bar{\hat{\alpha}}_1)$ | $\bar{\hat{\omega}}_0$ | $\bar{\hat{\sigma}}(\bar{\hat{\omega}}_0)$ | $\bar{\hat{\omega}}_1$ | $\bar{\hat{\sigma}}(\bar{\hat{\omega}}_1)$ | $\hat{\phi}_1$ | $\bar{\hat{\sigma}}(\hat{\phi}_1)$ | $\hat{\sigma}^2_{\hat{\varepsilon}}$ |
|---|---|---|---|---|---|---|---|---|---|
| GLS | 0.4355 | 0.0470 | 0.2959 | 0.0078 | 0.0344 | 0.0173 | 0.6494 | 0.0059 | 1.26743 |
| EML | 0.5066 | 0.1030 | 0.2973 | 0.0078 | 0.0845 | 0.0092 | 0.4872 | 0.1051 | 1.3403 |
| NLS | 0.4256 | 0.0561 | 0.2976 | 0.0074 | 0.0215 | 0.0105 | 0.5244 | 0.0956 | 1.2879 |

5.

**Table 3: Simulation results for** $\alpha_1 = 0.4$ , $\omega_0 = 0.3$ , $\omega_1 = 0.02$ , $\phi_1 = 0.5$ , $N = 120$

| Methods | $\bar{\hat{\alpha}}_1$ | $\bar{\hat{\sigma}}(\bar{\hat{\alpha}}_1)$ | $\bar{\hat{\omega}}_0$ | $\bar{\hat{\sigma}}(\bar{\hat{\omega}}_0)$ | $\bar{\hat{\omega}}_1$ | $\bar{\hat{\sigma}}(\bar{\hat{\omega}}_1)$ | $\hat{\phi}_1$ | $\bar{\hat{\sigma}}(\hat{\phi}_1)$ | $\hat{\sigma}^2_{\hat{\varepsilon}}$ |
|---|---|---|---|---|---|---|---|---|---|
| GLS | 0.3756 | 0.0432 | 0.2989 | 0.0046 | 0.0140 | 0.0127 | 0.6182 | 0.0103 | 1,0266 |
| EML | 0.5324 | 0.0793 | 0.2919 | 0.0048 | 0.0746 | 0.0052 | 0.5060 | 0.1047 | 1.5021 |
| NLS | 0.4322 | 0.0345 | 0.2948 | 0.0056 | 0.0179 | 0.0123 | 0.5134 | 0.0201 | 1.0341 |

**Table 4 : Simulation results for** $\alpha_1 = 0.4$ , $\omega_0 = 0.3$ , $\omega_1 = 0.02$ , $\phi_1 = 0.5$ , $N = 156$

| Methods | $\bar{\hat{\alpha}}_1$ | $\bar{\hat{\sigma}}(\bar{\hat{\alpha}}_1)$ | $\bar{\hat{\omega}}_0$ | $\bar{\hat{\sigma}}(\bar{\hat{\omega}}_0)$ | $\bar{\hat{\omega}}_1$ | $\bar{\hat{\sigma}}(\bar{\hat{\omega}}_1)$ | $\hat{\phi}_1$ | $\bar{\hat{\sigma}}(\hat{\phi}_1)$ | $\hat{\sigma}^2_{\hat{\varepsilon}}$ |
|---|---|---|---|---|---|---|---|---|---|
| GLS | 0.3892 | 0.0292 | 0.2991 | 0.0029 | 0.0180 | 0.0083 | 0.5566 | 0.0204 | 1.0488 |
| EML | 0.4439 | 0.0527 | 0.2868 | 0.0028 | 0.0177 | 0.0032 | 0.5439 | 0.0527 | 1.0298 |
| NLS | 0.4029 | 0.0345 | 0.2675 | 0.0021 | 0.0198 | 0.0041 | 0.5123 | 0.0205 | 1.0256 |

The second simulations concern a MISO model of the form

$$(1-\alpha_1 B)y(t) = (\omega_0 + \omega_1 B)x(t) + \frac{1}{(1-\phi_1 B - \phi_2 B^2)}\varepsilon(t).$$

**Table 5: Simulation results for** $\alpha_1 = 0.4$ , $\omega_0 = 0.5$ , $\omega_1 = 0.08$ , $\phi_1 = 0.5$ , $\phi_2 = -0.2$ , $N = 40$

| Methods | $\bar{\hat{\alpha}}_1$ | $\bar{\hat{\sigma}}(\bar{\hat{\alpha}}_1)$ | $\bar{\hat{\omega}}_0$ | $\bar{\hat{\sigma}}(\bar{\hat{\omega}}_0)$ | $\bar{\hat{\omega}}_1$ | $\bar{\hat{\sigma}}(\bar{\hat{\omega}}_1)$ | | $\bar{\hat{\sigma}}(\hat{\phi}_1)$ | $\hat{\phi}_2$ | $\bar{\hat{\sigma}}(\hat{\phi}_2)$ | $\hat{\sigma}^2_{\hat{\varepsilon}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| GLS | 0.352 | 0.0807 | 0.502 | 0.0118 | 0.0553 | 0.0414 | 0.8395 | 0.0275 | -0.4416 | 0.0191 | 1.1621 |
| EML | 0.401 | 0.2112 | 0.510 | 0.0098 | 0.1102 | 0.0164 | 0.6224 | 0.1980 | -0.4239 | 0.1880 | 1.8720 |
| NLS | 0.389 | 0.0765 | 0.501 | 0.0087 | 0.0754 | 0.0176 | 0.5145 | 0.0207 | -0.2435 | 0.0210 | 1.2089 |


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**Table 6: Simulation results for** $\alpha_1 = 0.4$ , $\omega_0 = 0.5$ , $\omega_1 = 0.08$ , $\phi_1 = 0.5$ , $\phi_2 = -0.2$ , $N = 80$

| Methods | $\bar{\hat{\alpha}}_1$ | $\bar{\hat{\sigma}}(\bar{\hat{\alpha}}_1)$ | $\bar{\hat{\omega}}_0$ | $\bar{\hat{\sigma}}(\bar{\hat{\omega}}_0)$ | $\bar{\hat{\omega}}_1$ | $\bar{\hat{\sigma}}(\bar{\hat{\omega}}_1)$ | $\hat{\phi}_1$ | $\hat{\phi}_1$ $\bar{\hat{\sigma}}(\hat{\phi}_1)$ | $\hat{\phi}_2$ | $\bar{\hat{\sigma}}(\hat{\phi}_2)$ | $\hat{\sigma}^2_{\hat{\varepsilon}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| GLS | 0.4991 | 0.0403 | 0.4961 | 0.0079 | 0.1329 | 0.0212 | 0.7189 | 0.0168 | -0.1687 | 0.0110 | 1.6053 |
| EML | 0.2341 | 0.1175 | 0.5011 | 0.0087 | 0.1005 | 0.0091 | 0.7759 | 0.1725 | -0.3715 | 0.1429 | 1.3988 |
| NLS | 0.4109 | 0.0356 | 0.5056 | 0.0076 | 0.0905 | 0.0198 | 0.5074 | 0.0123 | -0.1867 | 0.0213 | 1.4010 |

6.

**Table 7: Simulation results for** $\alpha_1 = 0.4$ , $\omega_0 = 0.5$ , $\omega_1 = 0.08$ , $\phi_1 = 0.5$ , $\phi_2 = -0.2$ , $N = 120$

| Methods | $\bar{\hat{\alpha}}_1$ | $\bar{\hat{\sigma}}(\hat{\alpha}_1)$ | $\bar{\hat{\omega}}_0$ | $\bar{\hat{\sigma}}(\bar{\hat{\omega}}_0)$ | $\bar{\hat{\omega}}_1$ | $\bar{\hat{\sigma}}(\bar{\hat{\omega}}_1)$ | $\hat{\phi}_1$ | $\bar{\hat{\sigma}}(\hat{\phi}_1)$ | $\hat{\phi}_2$ | $\bar{\hat{\sigma}}(\hat{\phi}_2)$ | $\hat{\sigma}^2_{\hat{\varepsilon}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| GLS | 0.3859 | 0.0341 | 0.4996 | 0.0040 | 0.0735 | 0.0170 | 0.5169 | 0.0120 | -0.0746 | 0.0079 | 1.0553 |
| EML | 0.4560 | 0.0579 | 0.4918 | 0.0050 | 0.0878 | 0.0065 | 0.6545 | 0.0048 | -0.2540 | 0.1316 | 1.2609 |
| NLS | 0.4089 | 0.0243 | 0.4987 | 0.0043 | 0.0765 | 0.0076 | 0.5087 | 0.0043 | -0.0672 | 0.0067 | 1.1342 |

7.

**Table 8: Simulation results for** $\alpha_1 = 0.4$ , $\omega_0 = 0.5$ , $\omega_1 = 0.08$ , $\phi_1 = 0.5$ , $\phi_2 = -0.2$ , $N = 156$

| Methods | $\bar{\hat{\alpha}}_1$ | $\bar{\hat{\sigma}}(\bar{\hat{\alpha}}_1)$ | $\bar{\hat{\omega}}_0$ | $\bar{\hat{\sigma}}(\bar{\hat{\omega}}_0)$ | $\bar{\hat{\omega}}_1$ | $\bar{\hat{\sigma}}(\bar{\hat{\omega}}_1)$ | $\hat{\phi}_1$ | $\bar{\hat{\sigma}}(\hat{\phi}_1)$ | $\hat{\phi}_2$ | $\bar{\hat{\sigma}}(\hat{\phi}_2)$ | $\hat{\sigma}^2_{\hat{\varepsilon}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| GLS | 0.3954 | 0.0227 | 0.4999 | 0.0029 | 0.0778 | 0.0109 | 0.4756 | 0.0069 | -0.0554 | 0.0029 | 1,0252 |
| EML | 0.4110 | 0.0125 | 0.4851 | 0.0038 | 0.0814 | 0.0040 | 0.5601 | 0.1225 | -0.2267 | 0.0903 | 1,1642 |
| NLS | 0.4023 | 0.0123 | 0.4906 | 0.0032 | 0.0811 | 0.0051 | 0.5009 | 0.0567 | -0.2076 | 0.0766 | 1.0891 |

The different results for the MISO models given in the above tables show that the GLS and the proposed algorithm (NLS) provide the values of the parameters which are close to those of the EML. A better performance can be attributed to the EML and the proposed algorithm which provide the parameter values that are close to the true values when the sample size increases. Also the EML and the NLS methods have given small standard errors of the parameters. The use of the optimization algorithm for the EML and NLS methods gives an advantage for these methods when the parameters increase. We have observed that for short time series ( $N = 40$ and $N = 80$ ), the EML is very sensitive to the quality of initial values of the parameters, it cannot always converge to the true values, particularly with improper initial values (Choi, 1992).

## 4. Conclusion

This paper has presented an algorithm for the iterative estimation of parameters of a MISO model. This algorithm is a generalization of that proposed by Hannan and Rissanen (1982). The EML method used for these simulation results has been proposed by Mélard (1984) which is a combination of an improved version of an algorithm of Pearlman (1980) that consists to replace the (matrix) Riccati-type difference equation used in the Kalman filter by a (vector) Chandrasekhar-type difference equation with the quick recursion switching suggested by Gardner, Harvey and Phillips (1980) and an algorithm of Wilson (1979). The comparison with the EML and the GLS methods from the simulation results show the good performance of the proposed algorithm.

**Conflict of Interest:** The authors declare no conflict of interest.

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